LOGARITHMS

In the mathematical world, algebra holds a large proportion. If we peak into algebra there are many formulas and rules to add or subtract powers. One of them is logarithms. In this guide, you will learn about logarithms, rules of logarithms, and how to use it.

What is Logarithm?

In simple words, the logarithm is a mathematical operation through which you can find the number of times a certain number is multiplied by itself. That certain number is called the base. One of the easiest examples is folding paper.

Paper Folding Example

Logarithms characterize the number of times you fold a sheet of paper in order to get 64 layers. When you fold a sheet of paper in half, you are just doubling the number of layers. In the mathematical language, the double layers mean 2 (the base) and we need to find how many times it needs to be multiplied by itself to get 64. This will be written as:

$$\log_2(64) = x$$

Since logarithm is the inverse of an exponential, we can write as:

$$2^x = 64$$

Let's solve it, how we can find the value of x.

$$2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$$

$$2^6 = 64$$

To get **64**, we need to multiply **2** by itself **6 times**. In conclusion, you need to fold the paper **six times** then you will get **64 layers**. In case if you don't understand, don't worry below is the detailed method to solve logarithms.

How to Solve Logarithm

Before we start how to solve logarithm, you should know how to convert log into an exponential term.

$$\log_b(a) = c \Longleftrightarrow b^c = a \qquad a > 0 \;\&\; a \neq 1$$

Where,

a is the argument,

b is the base, and

c is the exponent.

Through logarithms, you can find exponents. Let's just assume that you want to find the exponent of number **3** which is equal to **81**. So how will you write? You will write like this:

$$3^x = 81$$

Now you need to convert it into the log. Do check it twice or thrice that you place the argument, exponent, and base in the right position. Your answer should be:

$$\log_3(81) = x$$

$$x = 4$$

You can solve the log with the help of scientific calculators. These calculators have a logarithm function. All you need to do is press that button and enter your values and press the **"="** button. This feature is only available in scientific calculators so ensure that you have a scientific calculator.

The above example is easy to solve, now let's try some complicated problems. Don't get scared of the question! Sometimes questions look scary but inside they are very easy. Let's try to find the base this time.

$$\log_x(20) = 3$$

The question says to find the **base** which has the exponent **3** and it is equal to **20**. In the previous example, we converted our algebraic equation to the logarithmic equation. This time, we need to convert it to the algebraic equation and then we can put cube root on both sides to find the value of "x".

Please do note that you can't solve the log because it contains "x", why don't you try to solve it in a calculator? Does it give you syntax error? That is because of that unknown value. Therefore, we learn another thing and that is when there is a variable in the log, you can't solve it. To solve it, you need to convert it back to the algebraic equation and then find the value of x.

$$x^3 = 20$$

$$\sqrt[3]{(x^3)} = \sqrt[3]{(20)}$$

$$x = 2.714$$

Last but not least, sometimes log answer can be negative or in decimal. Don't panic if you answer comes negative or in small decimal numbers, you might be doing it correctly! Be confident!

Other Rules of Logarithm

Until now we have been doing the basics. There are rules for applying logarithm functions. You need to follow these rules to find the correct answer. The rules are not hard, if you keep practicing them, you will master it, after all, practice makes a man perfect!

Some rules are pretty basic, they need to be memorized but there are few rules that are conceptual. Let's first talk about those rules which are simple and you can easily memorize.

$$\log_b(1) = 0$$ (Logarithm zero rule)

$$\log_b(b) = 1$$ (Logarithm identify rule)

$$\log_b(b^n) = n$$ (Logarithm exponent rule)

Pretty simple right? The logarithm zero rule says that if the argument is 1 then your answer(exponent) will be 0. The logarithm identifies rule says if your base and argument are the same, your answer will be 1 (no matter what value you pick). Logarithm exponent rule tells if your argument has an exponent, the answer will be equal to that exponent.

There is a condition for the logarithm exponent rule, check carefully, base and argument are equal to each other. This means if argument and base are equal and if the argument has power then your answer will be equal to that exponent.

There are a few cases where you can't use the logarithmic function. If you tried to use it, the answer will be not possible. Here are those cases:

$$\log_{-b}(a)$$

$$\log_b(-a)$$

$$\log_b(0)$$

If your base is negative, the argument is negative, or your argument is equal to 0 then you can't solve it with logarithm function. This will help you by saving your time, you will already know the answer will not be possible.

Now we will talk about the rules which contain logic. These rules are important to know otherwise you will end up with the wrong answer. Below are the rules with their concepts.

$$\log_b(x.y) = \log_b(x) + \log_b(y)$$ (Logarithm product rule)

$$\log_b(x \div y) = \log_b(x) - \log_b(y)$$ (Logarithm quotient rule)

$$\log_b(x) = \log_c(x) \div \log_c(b)$$ (Logarithm base change rule)

It might look complicated but, as a matter of fact, it isn't. The logarithm product rules state that the log will split into 2 logs with the same base and both will be added. The logarithm quotient rule is almost the same but the difference is that when the argument is in fraction form, we will subtract the denominator log from the numerator log. The logarithm base change rule is a bit different. This rule states to change the base and then divide. First of all, write the log with an argument first and then divide it by log with base. You have the freedom to choose the base while dividing.

Examples

Logarithm product rule example:

$$\log_5(2x7)$$

$$\log_5(2x7) = \log_5(2) + \log_5(7)$$

$$\log_5(2x7) = 0.430 + 1.209$$

$$\log_5(2x7) = 1.639$$

Logarithm quotient rule example:

$$\log_3(4 \div 8)$$

$$\log_3(4 \div 8) = \log_3(4) - \log_3(8)$$

$$\log_3(4 \div 8) = 1.261 - 1.892$$

$$\log_3(4 \div 8) = -\ 0.631$$

Logarithm base change rule example:

$$\log_7(12)$$

$$\log_7(12) = \log_2(12) \div \log_2(7)$$

$$\log_7(12) = 3.584 \div 2.807$$

$$\log_7(12) = 1.276$$

(Do remember that in logarithm base change rule, you can choose any base you want, it can be 2 or 1000, the answer will be same)